Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

PUBLICATIONS

OF THE

Astronomical Society of the Pacific.

VOL. XI. SAN FRANCISCO, CALIFORNIA, DECEMBER 1, 1899. NO. 71.

A REVIEW.

BY SIDNEY D. TOWNLEY.

Untersuchungen über die astronomische Refraction, mit einer Bestimmung der Polhöhe von München und ihrer Schwankungen, vom November, 1891, bis October, 1893, und ein Katalog der absoluten Declinationen von 116 Fundamental-Sternen. VON DR. JULIUS BAUSCHINGER, *Neue Annalen der K. Sternwarte in München, Band III, 1898.*

In presenting the theory of refraction to a class of advanced students in practical astronomy last spring, the above investigation was reviewed in considerable detail, as an illustration of the latest methods and results in the subject of refraction. The subject is a very important one, and although numerous investigations have already been made, and accurate and convenient tables have been computed, still it has not reached the completeness demanded by the astronomy of precision of to-day. As interesting conclusions, some of which have a direct bearing upon the construction of meridian-circle houses, were reached, it has been thought that a review of the memoir may not be without interest to astronomical readers.

The first six articles are given up to a description of the instrument and an investigation of its circle, microscopes, etc.

The instrument used was a new Repsold meridian-circle, with an objective of 160^{mm} (about 6.3 inches).

The purpose of the work was to thoroughly investigate, as far as circumstances would permit, all questions of astronomical refraction. Inseparably connected with this was a new determination of the latitude and the compilation of a catalogue of fundamental declinations of a list of fixed stars. As to the usefulness of such a work, the author remarks (article VII, page 59): "The utility of such an investigation cannot be doubted; for, aside from the fact that an investigation of the refraction in Munich has never been made, and observatories in which fundamental measurements are to be carried out can scarcely longer forego this work, the excellence of the new instrument also promised a higher degree of accuracy than was obtained by BESSEL in his investigation made seventy years previously. The experiences of the last decades have so often pointed to the necessity of change in the fundamentals adopted by BESSEL, that, from the present standpoint of instrumental equipment and accuracy of observing, a thoroughgoing investigation seems necessary, in order to obtain the greatest possible control over this most disturbing factor in astronomical observations. Besides this, advances which have not yet been utilized have been made in the mathematical treatment of refraction, as well as our knowledge of the atmosphere; and further, several questions of detail, such as the influence of the temperature of the observing-room, the humidity of the atmosphere, etc., seem hardly to have been sufficiently investigated."

The author makes here no reference to several investigations more recent than those of BESSEL.

The stars used in the investigation were taken from AUWERS'S "Fundamental Catalogue," in order to have a control upon the positions deduced, and also to lessen the labor of reduction, by using the proper motions there deduced. Polar stars were observed, at both upper and lower culmination, in all positions from the zenith to the northern horizon. Some stars culminating south of the zenith were also observed. In order to concentrate all the attention upon zenith-distances, no observations of Right Ascension were made. Something over four thousand observations were secured, on a list of two hundred and one stars, counting lower culmination as a separate star. Six hundred and fifty-eight culminations at a zenith-distance greater than 80°, and two hundred and seventy at a zenith-distance greater than 85°, were observed.

In the eighth article the meteorological data are fully discussed. In an investigation of this nature the manner of taking and working up the meteorological observations is a matter of the greatest importance. The first reduction of the zenith-distance observations was made by means of the RADAU'S tables ("Essai sur les Réfractions Astronomiques," *Memoires de l' Observatoire de*

Paris, t. XIX), which are convenient of application and accurate for large zenith-distances. The treatment of the meteorological observations was made along lines suggested in this memoir.

The barometer-readings besides being corrected for temperature of the mercury, the change in gravity with the latitude and the height above sea-level, were also corrected for the humidity of the atmosphere in the following way. FIZEAU and JAMIN have shown that the index of refraction for moist air is less than that of dry air, and such as would be produced by reducing the density in the ratio of 1 to $1 - \frac{1}{8}\frac{\pi}{760}$, where π is the absolute water-vapor tension in millimeters. If the index of refraction of dry air were determined and lay at the foundation of the tables, then for a vapor-tension of π the correction to the refraction could be brought about by a correction to the barometer-reading of $-\frac{1}{8}\frac{\pi}{760}B$. RADAU used, however, the index of refraction determined by BESSEL at Königsberg under a mean vapor-tension of 6^{mm}; hence the barometer-readings require only the correction $\frac{6-\pi}{8}\frac{B}{760}$.

The outside temperature was measured by means of a mercury centigrade thermometer hung in an open metal house outside the north window of a room near the meridian-circle room. The thermometer stood one meter from the wall and two and one-tenth meters from the ground. Toward the latter part of the observations an "Assmann'sches Aspirationsthermometer" (a thermometer past which a current of air is continually drawn) was used also, but no systematic difference was found between its readings and those of the other thermometer.

The temperature of the observing-room was very carefully taken by means of five thermometers suspended near the meridian-plane of the instrument. These were always read at the same time as the outside thermometer. It was found that the southern part of the room was warmer than the northern, which is to be explained probably by the greater heating of the southern walls (stone or brick). An investigation of all the readings of two years gave the following differences between inside and outside temperatures, taken in the sense inside minus outside:—

Nights		+ 1°.3
Mornings—	Winter	+ 1 .3
"	Summer	— 0 .4
Afternoons—	Winter	+ 1 .0
"	Summer	+ 0 .4

The differences were least with a strong wind, and greatest in calm weather.

The direction and velocity of wind and wet-bulb thermometer were also observed.

Article X is given up to the determination of the latitude and its variation. This can be done easily with a meridian-circle by observations of polar stars at upper and lower culmination, provided the division-errors of the circle have been well determined. For the Munich instrument this was done only for the 5° marks; and as the errors appeared to be purely accidental, it was impossible to interpolate for other divisions. A first approximation to the latitude was found from the numerous observations of α *Ursa Minoris* and *1 H Draconis*. The final value of the latitude, however, and its variation were only found after a number of approximations in which were used all of the stars which were observed at both upper and lower culmination.

With the value of the latitude and its variation so deduced, a table of preliminary declinations was derived from the observed zenith-distances.

The mean errors of the observations are derived in Article XII.

We have considered thus far only preliminary operations,—the preparation of the material for the discussion of the questions which it was proposed to investigate. The first taken up, Article XIII, is the coefficient of expansion of the air and the influence of vapor-tension. Some interesting results were derived.

In RADAU'S tables, REGNAULT'S value of the coefficient of expansion is used, namely:—

$$a = \frac{1}{273} = 0.003663 \text{ (for centigrade).}$$

The following investigation seemed to lead, at first, to a considerable correction. For this only stars between zenith-distances of 60° and 85° north were used, because those of a smaller zenith-distance can bring only a minimum consideration to the solution of the problem, while those of a large zenith-distance involve perturbations of such magnitude as to swallow up any influence from small changes of temperature.

For each star the mean of the four zenith-distances at the highest temperature were taken; likewise the mean of the zenith-distances at the four lowest temperatures; likewise the mean of the temperatures. The differences of temperature are for the most part large, and valuable results should be expected. The differences $z_1 - z_0$ are in general but little larger than their mean errors, but the greater portion of them are positive, thirty-five out of forty-five. The cause of this discrepancy was sought in the coefficient of expansion. If $1 + \frac{i}{100}$ is the factor by which the coefficient of expansion, 0.003663, is to be multiplied, and if R is the refraction for 0° C. and 718^{mm} barometer, then, neglecting terms of the second order, we have observation-equations of the following form:—

$$(t_1 - t_0)\, 0.003663\, \frac{R}{100}\, i = z_1 - z_0$$

Forty-five equations combined by the Methods of Least Squares gave

$$i = +\, 3.19 \pm 0.91$$

and hence for the coefficient of expansion,—

$$0.003663\, (1 + 0.0319) = 0.003780 \pm 0.000033$$

This increase of three per cent. seems inadmissible, notwithstanding the fact that other investigators have found equally large results. For instance,—

Gyldén,	0.003769,	Obs. de Poulkova, t. II, p. (16)
Nyrén,	0.003770,	" " " Ser. II, t. I, p. (2)
Mascart,	0.00382	Ann. d'Ecol. norm., 2. S. VI, 9

Neglecting the vapor-tension made the differences $z_1 - z_0$ "more positive." This emphasized the necessity of taking the vapor-tension into account, and also seemed to indicate that the vapor-tension factor had not been correctly computed. As already pointed out, Radau used the expression

$$\left(1 - \tfrac{1}{8}\, \frac{\pi}{760}\right) B \text{ instead of } B$$

where the empirical factor ⅛ is taken from the experiments of Fizeau and Jamin, and depends upon the so-called optical density of the air. Theoretically, this distinction between optical and physical density is justified, and the only question is, Is this optical density to be used in astronomical observations where the conditions are essentially different from those of an experiment in the physical laboratory? To answer this, the coefficient of π was computed directly from the observations. This leads us back to the differences $z_1 - z_0$, because in general the extremes of temperature correspond to the extremes of vapor-tension.

If $\frac{k}{8}$ is the factor to be determined, then the equation is of the form

$$0.12\, m\, (\pi_1 - \pi_0)\, (k - 1) = z_1 - z_0$$

where m means the change of refraction for 1^{mm} of mercury pressure (to be taken direct from RADAU'S tables). The solution of the forty-five equations gave

$$k - 1 = 3.37 \pm 0.69$$

The observations then decide in favor of a coefficient of π, *which proves that the refraction depends upon the physical density of the air.*

This is known, namely:—

$$\left(1 - 0.378 \frac{\pi}{760}\right) B, \text{ or nearly } \left(1 - \tfrac{3}{8} \frac{\pi}{760}\right) B$$

The remaining errors bear no systematic relation either in an arrangement according to zenith-distance or Right Ascension, from which was inferred, *that after the introduction of the new coefficient of* π, *the observations do not demand a change in the adopted value of the coefficient of expansion.*

The constant of refraction (Article XIV) was deduced from the declinations obtained from upper and lower culminations, without reference to the conclusions reached in the preceding article. Let

δ = declination from upper culmination
δ' = " " lower "
r = refraction for upper "
r' = " " lower "
$\Delta\,\phi$ = correction to the latitude employed
$1 + n$ = the factor by which the refraction employed (BESSEL'S constant) is to be multiplied in order to satisfy the observations at hand;

Then

$$\delta - \delta' = -2\,\Delta\,\phi \pm r\,n - r'\,n \begin{cases} \text{upper cul. north of zenith} \\ \text{'' '' south '' ''} \end{cases}$$

Putting $-2\,\Delta\,\phi = x$, $-100\,n = y$, then

$$\delta - \delta' = x + y\,\frac{\pm r + r'}{100} \begin{cases} \text{upper culmination north of zenith} \\ \text{'' '' south '' ''} \end{cases}$$

From the observation data, weighted observation-equations were formed by this formula, for all stars observed at both upper and lower culmination. The coefficient $\frac{\pm r + r'}{100}$ was taken direct from the values of the refraction employed.

The solution of these equations gave

$$\left.\begin{array}{lll} x = -\,0''.797 & \text{weight} & 163.08 \\ y = +\,0\ .510 & \text{``} & 1053.92 \\ [P\Delta\Delta] = 140''.18 & & \end{array}\right\}\ \text{I.}$$

The considerable reduction in the value of Bessel's constant of refraction, indicated in the large value of y, must be taken with "*Vorsicht,*" notwithstanding its large weight and the fact that other recent investigations have given similar results. An inspection of the observation equations showed that if the observations are taken only to 76° zenith-distance, no appreciable change would result in either the latitude or constant of refraction. Such a solution gave:—

$$\left.\begin{array}{lll} x = -\,0''.047 & \text{weight} & 29.79 \\ y = -\,0\ .028 & \text{``} & 70.52 \\ [P\Delta\Delta] = 48''.90 & & \end{array}\right\}\ \text{II.}$$

The equations below 76° zenith-distance gave

$$\left.\begin{array}{lll} x = -\,0''.575 & \text{weight} & 19.30 \\ y = +\,0\ .483 & \text{``} & 323.28 \\ [P\Delta\Delta] = 67''.87 & & \end{array}\right\}\ \text{III.}$$

It was found impossible to reconcile these last two solutions, and the inference is that there is some important factor, affecting the observations at large zenith-distances, not yet taken into account. It was thought that this discrepancy might lie in the law of decrease in the temperature of the atmosphere, by which undoubtedly the observations below 76° zenith-distance are influenced, while it is known that the refraction for stars of a smaller zenith-distance than 76° is practically the same, no matter what law of the constitution of the atmosphere is assumed. Radau's tables are computed upon Ivory's hypothesis with the factor $f = 0.2$. If the true value of f be taken to be $\frac{2 + z}{10}$, then the change in the refraction will be $-\,\lambda\, z$, in which λ can be taken from table V, Radau, and the observation equations then take the form—

$$\delta - \delta' = x + y\,\frac{\pm r + r'}{10} + \lambda\, z$$

The solution of these equations gave

$$\left.\begin{array}{lll} x = -\,0''.828 & \text{weight} & 126.2 \\ y = +\,0\ .527 & \text{``} & 672.1 \\ z = -\,0\ .053 & \text{``} & 188.6 \\ [P\Delta\Delta] = 138''.65 & & \end{array}\right\}\ \text{IV.}$$

The introduction of z did not materially better the situation; moreover, the small value of z shows that no appreciable change in $f = 0.2$, is demanded by the observations.

It was then thought than an explanation of the discrepancy might lie in the fact that at great zenith-distances the star-image is drawn out into a spectrum, and we can get from these observations an index of refraction for white light, which obtains for stars at small zenith-distances, only when a particular part of the spectrum is set upon. In order to investigate this, the index of refraction was determined from solutions II and III, as follows:—

The observations were made with a mean barometer 718^{mm} and a mean temperature of $+ 5^\circ$ C.; for these BESSEL's constant of refraction, which has been used in the tables employed, is $56''.076$.

$$\begin{array}{ll} \text{From II} & +0.00028 \times 56''.076 = +0.016 \\ \text{“ III} & -0.00483 \times 56\ .076 = -0.271 \end{array}$$

whence we have for the two cases $56''.092$, $55''.805$. These quantities, for pressure 760^{mm} and temperature 0° C., are $60''.461$ and $60''.149$, and the corresponding indices of refraction are—

II. 1.0002933
III. 1.0002918

All these numbers correspond to a vapor-tension of 6^{mm}. KAYSER and RUNGE, through laboratory investigations, have found for the principal Fraunhofer lines (pressure 760^{mm}, temperature 0° C., vapor-tension 6^{mm}).

A = 1.0002902
B = 1.0002908
C = 1.0002911
D = 1.0002919
E = 1.0002930
F = 1.0002940

The value II above lies in the green, while the value III, for stars at great zenith-distances, lies between the red and yellow; a result to be expected on account of atmospheric absorption. The conclusion is that stars culminating at great zenith-distances are not suitable for the determination of the constant of refraction. Stars to about 80° can be used. Very small zenith-distances must also be used; otherwise the separation of x and y will be impossible. With these assumptions, another solution was made, and gave

$$\left.\begin{array}{rlrl} x = & -0''.685 & \text{weight} & 58.79 \\ y = & +0\ .442 & \text{“} & 189.43 \\ [P\Delta\Delta] = & 82''.92 & & \end{array}\right\} \text{V}$$

This is nearly the solution I, and not more satisfactory.

As far as BAUSCHINGER could see, there was only one other possible way of explaining away these discrepancies, and that was through the *difference in temperature between the observing-room and the outside air.* This was investigated as follows :—

From the theory of refraction —

$$R = \int_1^{\mu_0} \frac{\sin z}{\sqrt{\dfrac{\mu\, r}{\mu_0\, r_0} - \sin^2 z}} \, \frac{d\,\mu}{\mu} \qquad \left(\begin{array}{l} \mu = \text{index of refraction.} \\ r = \text{distance of layer from} \\ \quad\;\; \text{center of Earth.} \end{array} \right.$$

Putting $\frac{\mu\, r}{\mu_0\, r_0} = 1$, we have $(R) = \tan z \int_1^{\mu_0} \frac{d\,\mu}{\mu} = \tan z \log. \text{nat.}\, \mu_0$

which shows that the refraction depends almost entirely upon the condition of the atmosphere in the lowest layer of the air. Where is this last layer? *In the observing-room*, just before the rays enter the telescope. The inference is that the inside, rather than the outside, temperature must be used. This, however, cannot be done so easily, because at the limit of the room the parallelism of the layers is destroyed. Through the following process an approximation can be reached: Conceive two layers, one corresponding to the outer temperature, one to the inner. At the dividing-line between them a new refraction takes place. To compute this it is necessary to know the form of the dividing-surface. Only when it is a cylinder whose axis is coincident with the rotation-axis of the instrument would no refraction take place. This in general will not be the case. The limit will correspond to the outline of the observing-room, and we must assume that the inner temperature is largely controlled by the radiation of the walls. If we assume this, then there will be two cases : one when the rays come in the top opening, the other when they enter through the side opening. The upper opening can be considered parallel to the general layers, and the refraction here will take place according to the same laws as in general. Let z' and z_0 be the zenith-distance of the outer and the measured rays ; μ' and μ_0 the index of refraction for outer and inner air ; ρ' and ρ_0 the densities ; then we have, according to the law of SNELLIUS,—

$$\frac{\sin z'}{\sin z_0} = \frac{\mu_0}{\mu'} = \sqrt{\frac{1 + 2\, c\, \rho_0}{1 + 2\, c\, \rho'}} \, ;$$

or, $$\frac{\sin^2 z_0 - \sin^2 z'}{\sin^2 z_0} = \frac{2\, c\, \rho'}{1 + 2\, c\, \rho'} - \frac{\rho_0}{\rho'} \, \frac{2\, c\, \rho'}{1 + 2\, c\, \rho'} = 2\, \alpha' \left(1 - \frac{\rho_0}{\rho'}\right),$$

where $\alpha' = \dfrac{c\,\rho'}{1+2\,c\,\rho'}$ is the constant of refraction.

If we put further $z_0 - z' = R_s$ and

$$1-\frac{\rho_0}{\rho'} = \frac{m\,(t_0-t')}{1+m\,t_0}\left\{\begin{array}{l} m = \text{coefficient of expansion of the air} \\ t_0 = \text{inner temperature} \\ t' = \text{outer} \quad \text{''} \end{array}\right.$$

then we have, expanding and neglecting second and higher powers of R_s,—

$$R_s = \left(\frac{m\,\alpha'}{1+m\,t_0}\tan z_0\right)\left(t_0-t'\right)$$

The part in first brackets is the temperature coefficient, and we can thus compute the difference between outside and inside refraction.

For the side-walls the refraction is perpendicular to that just considered; and we therefore have

$\dfrac{\cos z'}{\cos z_0} = \dfrac{\mu_0}{\mu'}$ and in a way similar to that just employed

$$R_s = -\frac{\alpha'\,m}{1+m t_0}\operatorname{ctn} z_0 (t_0 - t') = -\left(\frac{\alpha'\,m}{1+m t_0}\tan z_0\right)\operatorname{ctn}^2 z_0 (t_0 - t')$$

If z = the true zenith-distance, the quantity ultimately sought,

z' = the opponent zenith-distance of the rays at opening of shutter,

R = refraction computed with outside temperature,

z_0 = measured zenith-distance,

R_s = refraction in observing-room given by above formulæ;

then, $z = z' + R$,

$z_0 - z' = R_s$, or $z' = z_0 - R_s$,

$z = z_0 + R - R_s$.

In the Munich meridian-circle room the dividing line between top and side openings is at about 60° zenith-distance, necessarily somewhat indeterminate on account of the thickness of the walls.

After applying the corrections R_s, and forming new quantities, δ-δ', a new solution from all stars was made, and gave the following results:—

$$\left.\begin{array}{rll} x = & -\,1''.018 & \text{weight } 119.2 \\ y = & +\,0\ .553 & \quad\text{''} \quad 603.1 \\ z = & +\,0\ .033 & \quad\text{''} \quad 90.5 \\ [P\,\Delta\,\Delta] = & 111.18 & \end{array}\right\}\text{VI.}$$

Using only stars to 76° zenith-distance, gave

$$\left.\begin{array}{rll} x = & -\,0''.912 & \text{weight } 29.79 \\ y = & +\,0\ .445 & \quad\text{''} \quad 70.52 \\ [P\,\Delta\,\Delta] = & 54.19 & \end{array}\right\}\text{VII.}$$

A comparison of these two sets shows that there is no longer a "*Widerspruch*" between the results from small and large zenith-distances. The observations to 76° zenith-distance are represented through VI just as well as by VII. The $[P \Delta \Delta]$ in the two cases being $55''.77$ and $54''.19$.

No other way of handling this temperature difference was apparent. Finally, then, to obtain the constant of refraction it was necessary to bring in the correction for water-vapor, which was shown to be necessary.

The refraction is to be computed with the expression, $B + \frac{k}{8} \cdot \frac{6 - \pi}{760} B$, instead of B.

The factor k was found to be 4.37 ± 0.69. Instead of this value, however, which was not determined with great certainty, the value obtained by laboratory methods (namely, $k = 3$) was used.

With the finally corrected values of $\delta - \delta'$ a new solution gave; All stars :—

$$\left.\begin{array}{rll} x = & -1''.036 & \text{weight } 157.86 \\ y = & +0\;.563 & \quad\text{"}\quad 999.43 \\ [P \Delta \Delta] = & 140.88 & \end{array}\right\} \text{VIII.}$$

Stars down to 76° zenith-distance :—

$$\left.\begin{array}{rll} x = & -0''.953 & \text{weight } 29.79 \\ y = & +0\;.491 & \quad\text{"}\quad 70.52 \end{array}\right\} \text{IX.}$$

The solution IX differs but little from VIII, and, on account of small weight, can be considered identical with it. VIII differs but little from VI. Solution VIII was considered final.

For 718^{mm} (at 0° C.) mercury pressure, + 5° C. temperature, and 6^{mm} vapor-tension, BESSEL'S constant of refraction is, according to RADAU'S tables, $56''.076$. This requires now the correction $56''.076 \times 0.00563 = -0''.315$, giving thus $55''.761$. This gives for 760^{mm} pressure, 0° C. mercury temperature, 0° C. air temperature, and 6^{mm} vapor-tension, $60''.104 \pm 0''.025$. The corresponding index of refraction for the air is $1.00029152 \pm 0.00000012$. The correction to the latitude is $\Delta \phi = +0.''518 \pm 0''.056$.

With the corrected value of latitude, the corrected constant of refraction, the corrections for refraction in observing-room, and humidity of atmosphere, a new table of definitive declinations was computed.

Another investigation of interest, although without theoretical foundation, was carried out. It was seen that the large differences between upper and lower culmination at great zenith-distances would disappear if a temperature about 1° higher had been used in computing the refraction. The temperature of the observing-room was in general about 1° higher than outside. The refraction was computed direct with the temperature of the observing-room, and again the differences $\delta - \delta'$ formed. The solution was again carried through, and gave:—

$$\left.\begin{array}{ll} x = -0''.419 & \text{weight } 157.86 \\ y = -0\ .194 & \text{`` } \quad 999.43 \end{array}\right\} \text{X.}$$

The outstanding errors, both in sign and magnitude, are very nearly the same as in VIII, so that this criterion will throw no light upon the question as to which is the correct method of procedure; but according to the above theoretical considerations, the solution X is to be thrown out. The work is not, however, entirely useless, because it shows to what an extent our declination systems are dependent upon the manner in which the temperature is measured.

BAUSCHINGER here states, as his opinion, with the emphasis of italics, that an absolute system of observations to be wholly free from objections must be made in the open air. The German loses much of its force in translation, and I therefore give the original statement.

"Meiner Ansicht nach ergibt sich hieraus die Forderung, *dass völlig einwandfreie absolute Systeme ganz in freier Luft beobachtet werden müssen.*"

Another interesting investigation bearing upon the law of the decrease of the temperature in the atmosphere was carried out.

It is sometimes thought that a knowledge of the temperature in the upper layers of the atmosphere can be obtained from astronomical observations. This is true only to a very limited extent. Before this law can be ascertained, one must determine with great accuracy many factors which enter into it — chief of which are the constant of refraction and its dependence upon the part of the spectrum pointed upon, the coefficient of expansion of the air, the humidity, and above all the temperature of the last layer of air. To separate the effect of each one of these factors is considerable of a problem, as RADAU'S extensive calculations have shown. BAUSCHINGER made some observations and calculations in the hope of throwing light upon the subject.

The observations can be satisfied by many different laws, when corresponding changes are made in the constant of refraction.

Comparison of IVORY'S hypothesis, as used by RADAU, and GYLDÉN'S hypothesis and a combination of the two, leads to the following conclusion, namely: that the observations are represented equally well by either hypothesis:—

IVORY'S hypothesis gives: $t_0 - t = 5^\circ.69\, h - 0^\circ{:}19\, h^2$;
GYLDÉN'S " " $t_0 - t = 5\ .10\, h - 0\ .025\, h^2$;

where t_0 is temperature of lowest layer, t temperature of the layer at h kilometers above the ground.

Meteorological observations in balloons bear directly upon this subject. Perhaps in the future such experiments may bring valuable results, but thus far they have given little else than an extraordinary changeableness in the "law" of decrease of temperature, in the layers to about 2^{km} height.

Observations were made in Bavaria, at night, by Professors SOHNCHE and FINSTERWALDER. For altitudes between 300 and 2,000 meters a decrease in temperature, 10° per km. in clear summer nights was found. Up to 300^{m} there is often, and always at night, an increase in the temperature.

As far as present evidence goes, it does not seem possible to improve on IVORY'S hypothesis, up to an altitude of 10^{km} at least. The difference between actual observed temperatures and those given by the hypotheses of IVORY and GYLDÉN follows. The highest reading was at 16^{km}. From 10 to 16^{km} IVORY'S hypothesis falls considerably short.

At 16^{km} Obser. — IVORY $= + 27^\circ$
" " — GYL. $= - 6$

GYLDÉN'S agreement is very good throughout. If these observations are confirmed by future experiments, IVORY'S hypothesis must be abandoned, although it will be sufficiently exact to zenith-distance of 88°.

As already mentioned, the temperature on clear nights reaches a maximum at some distance above the Earth. From balloon observations and observations upon the Eiffel tower in Paris, it was found that the increase was about 2° C., and the maximum reached at about 200^{m}.

It is interesting to know the effect of this condition upon the refraction. RADAU investigated this through an elegant analysis, which, however, leads to extensive calculations. The problem can be successfully handled by mechanical quadratures.

Consider the atmosphere divided into two parts, at the maximum temperature point. Assume the temperature to increase proportional to the distance in the first part, and to decrease according to IVORY'S law in the second part. The refraction in the second part can be computed from the ordinary tables, provided we know the height of the barometer at the altitude H, the separating line. This is easily found by the formula:—

$$H = 18401.2\left(1+\frac{T}{273}\right)\left(1+3/8\,\frac{\pi}{P}\right)\left(1+0\ 0026\cos 2\phi\right)\left(1+0.0000000196\,b\,log\,\frac{P}{p}\right)$$

where T is mean temperature of the layer,
" P is barometer height at ground,
" p is " " " H,
" b is height of ground above sea-level.

The refraction in the first part can be computed by

$$(R) = \int_{\mu_1}^{\mu_0} \tan z\,\frac{d\,\mu}{\mu}$$

The whole distance H is divided into layers, of say 10^{m} thickness.

Values of μ for every 10^{m} were computed, for certain atmospheric conditions, and then the processes of mechanical quadrature applied to the integral for certain special values of tan z.

The results are as follows:—

z_0	74° 2′	79° 4′	82° 16′	84° 7′	86° 22′	87° 56′
(R)	6″.08	9″.01	12″.82	16″.90	27″.44	48″.32

In upper part (from tables):—

$((R))$	3′7″.04	4′32″.87	6′18″.15	8′ 1″.74	11′48″.02	16′49″.55
R_a	3′13″.12	4′41″.88	6′30″.97	8′18″.64	12′15″.46	17′37″.87

From tables with lower temperature:—

R_n	3′13″.06	4′41″.65	6′30″.34	8′17″.32	12′10″.38	17′21″.51
R_a-R_n	+ 0″.06	+0″.23	+ 0″.63	+ 1″.32	+ 5″.08	+ 16″.36

From the values of $R_a - R_n$ it is seen:—

1. That down to about 80° zenith-distance, the effect on the refraction, of the observed temperature inversion, is negligible.

It has already been found through other investigations that refractions cannot be relied upon below this limit.

2. That the above values of $\delta - \delta'$ would be still greater, and consequently BESSEL'S refraction constant still more decreased, if we should take into account the effect of this inversion of the temperature.

It appears from all these investigations that no condition of the atmosphere, consistent with the observed meteorological conditions, will explain away the differences $\delta - \delta'$. There seems, then, no escape from the conclusion that BESSEL'S constant of refraction must be decreased. Several other recent investigations lead to the same conclusion. A comparison of its value is made with all the previous determinations.

In the above review I have attempted only once to give an exact translation. Throughout, however, I have followed closely the author's order of presentation, and in many places have used nearly the exact language of the original.

One who studies the memoir cannot help but to be impressed with the thoroughness of the investigation and with the scientific attitude of the investigator. Apparently he has spared no labor, neglected no detail in his effort to obtain the correct interpretation from the data of observation.

The most important conclusions reached in the investigation are as follows:—

1. In investigations of this nature, and in all meridian-circle observations made to determine accurate zenith-distances, the manner of observing the temperature is of the utmost importance.
2. The difference between inside and outside temperature cannot be neglected. It is apt to be considerable, especially in the older form of meridian-circle houses, and is greatest in the best observing weather — calm.
3. The effect upon the refraction of the observed inversion of the temperature at night is negligible to a zenith-distance of 80°.
4. In computing the refraction at the shutters the form of the opening must be taken into account. To be entirely free from objection, observations for an absolute system should be made in the open air.
5. The humidity of the air must be taken into account in correcting the barometer readings. Other recent investigations have led to the same conclusion.

6. The astronomical refraction depends upon the physical density of the air, and not, as before assumed, upon the optical density.

7. Granting 6, then the Munich observations do not demand a change in the adopted value of the coefficient of expansion of air. A result which agrees with other (not all) recent investigations, laboratory as well as astronomical.

8. The Munich observations, as well as others, show the necessity of a considerable decrease in BESSEL'S constant of refraction.

PROGRAMME OF THE INTERNATIONAL GEODETIC ASSOCIATION FOR OBSERVING VARIATIONS OF LATITUDES.

By FRANK SCHLESINGER.

The twelfth General Conference of the International Geodetic Association was held at Stuttgart, in October, 1898. A plan that had been under contemplation for several years for observing variations of latitudes was there formally adopted. Since then the observations have actually been begun and some of the details of the work may not be without interest.

I. SELECTION OF THE STATIONS.

The general aim of the association was to establish four stations on the same parallel of latitude, as well distributed in longitude as possible; to equip these stations with zenith-telescopes embodying some modifications that previous experience has indicated, and of larger aperture than usually employed; to so arrange the programme that all the observations at one station could be made by a single observer. It was of course necessary that each station should be located where there was a good average of clear nights, not only for the whole year, but for each season, in order to guard against a long interruption of the observations. It was deemed desirable that the character of the country should be the same to the north and to the south of each station, and that the seismological, hygienic, and social conditions should be good. With all these requirements to be considered,